Commonly Used Notations

Probability Notations

- $(\Omega, \mathcal{F}, \mathrm{P})$: probability space.
- $\mathcal{F}_t$, $\mathcal{B}_t$: filtrations of σ-algebras.
- P: probability measure.
- Q: risk-neutral measure.
- Q^B: spot Libor measure.
- Q^T, Q^n: forward measure for time T or T_n (given tenor structure).
- $\mathrm{Q}^{n,m}$: swap measure for swap rate $S_{n,m}$ (given tenor structure).
- Q^N: measure for numeraire N.
- E, E^{P}, E^{Q}, E^T, E^n, $\mathrm{E}^{n,m}$, E^A, ...: expectations under various measures.
- E_t, $\mathrm{E}_t^{\mathrm{P}}$, $\mathrm{E}_t^{\mathrm{Q}}$, E_t^T, E_t^n, $\mathrm{E}_t^{n,m}$, E_t^A, ...: expectations conditional on $\mathcal{F}_t$ under various measures.
- $Z(t)$, $W(t)$, $W^T(t)$, $W^n(t)$, $W^{n,m}(t)$, $W^A(t)$, ...: Brownian motions under various probability measures.
- $\mathrm{Var}(X)$: variance of X.
- $\mathrm{Stdev}(X)$: standard deviation of X.
- $\mathrm{Cov}(X, Y)$: covariance of X, Y.
- $\mathrm{Corr}(X, Y)$: correlation of X, Y.
- $\mathcal{N}(\mu, \Sigma)$: Gaussian distribution with mean μ and variance-covariance matrix Σ.
- $\mathcal{LN}(\mu, \sigma^2)$: log-normal distribution with mean μ and variance σ^2.
- $\mathcal{U}(a, b)$: uniform distribution on an interval $[a, b]$.
- $\Phi(z)$: standard Gaussian CDF, $\phi(z)$: standard Gaussian PDF.
- $\Gamma(a, x)$: the (upper) incomplete Gamma function, $\Gamma(a, x) = \int_x^\infty u^{a-1} e^{-u}\, du$.
- $\Gamma(a)$: the Gamma function, $\Gamma(a) = \Gamma(a, 0)$.
- $\mathcal{E}(X(t))$: Doleans exponential martingale for the process $X(t)$.
- $\langle X(t) \rangle$, $\langle X(t), Y(t) \rangle$: quadratic variation and covariation.

Finance Notations

- $T_0 < T_1 < \ldots < T_N$: tenor structure.
- τ_n: year fraction between T_n and T_{n+1}.
- $\beta(t)$: continuously compounded money market account.
- $B(t)$: discretely compounded money market account.
- $P(t,T)$: zero-coupon (or discount) bond price at time t for maturity T.
- $P(t,T,S)$: forward bond price at time t, for delivery of S-maturity discount bond at time T, $T \leq S$.
- $y(t,T,S)$: continuously compounded forward yield at time t for the period $[T,S]$.
- $f(t,T)$: instantaneous forward rate at t for maturity T.
- $r(t)$: short rate at time t, $r(t) = f(t,t)$.
- $L(t,T,S)$: forward Libor rate at time t for the period $[T,S]$.
- $L_n(t)$: forward Libor rate at t for the period $[T_n, T_{n+1}]$, given a tenor structure, $L_n(t) = L(t,T_n,T_{n+1})$.
- $S_{n,m}(t)$: forward swap rate at time t, starting at T_n and with the final payment date at T_{n+m} (given a tenor structure).
- $A_{n,m}(t)$: annuity at time t, with the first payment date T_{n+1} and the final payment date T_{n+m} (given a tenor structure).
- $U_n(t)$: the n-th exercise ("underlying") value of a Bermudan swaption or a callable Libor exotic.
- $H_n(t)$: the n-th hold value of a Bermudan swaption or a callable Libor exotic.
- $\sigma_{\mathrm{B}}(t,S;T,K)$: an implied Black volatility smile, parameterized by the time t spot S, strike K and expiry T.
- $c_{\mathrm{B}}(t,S;T,K)$, $c_{\mathrm{B}}(t,S;T,K,\sigma)$: price of a call option in the Black model with time t spot S, strike K, expiry T and Black volatility σ.
- $c_{\mathrm{N}}(t,S;T,K)$, $c_{\mathrm{N}}(t,S;T,K,\sigma)$: price of a call option in the Gaussian (Normal, or Bachelier) model with time t spot S, strike K, expiry T and Normal volatility σ.

Miscellaneous Notations

- $\mathrm{Re}\,(z)$, $\mathrm{Im}\,(z)$: real and imaginary part of a complex number z.
- $O(\cdot)$, $o(\cdot)$: "Big O" and "Little o" order symbols.
- $1_{\{A\}}$: indicator of A.
- L^1 and L^2: spaces of integrable and square-integrable random variables, vectors, or functions.
- C^n: space of functions with the n-th continuous derivative, i.e $C = C^0$ are continuous functions, C^1 are differentiable functions with continuous derivative, C^2 are twice-differentiable functions with continuous second-order derivative, etc.

- $\mathcal{L}$, $\mathcal{J}$: differential operators, e.g. $a\,\partial/\partial x + b\,\partial^2/\partial x^2$ or $\partial/\partial t + a\,\partial/\partial x + b\,\partial^2/\partial x^2$.
- $(\mathcal{F}f)(\omega), (\mathcal{F}^{-1}\varphi)(x)$: direct and inverse Fourier transforms.
- $\triangleq$: "is defined as", e.g. $f(x) \triangleq x^2$.
- x^+, x^-: maximum and minimum of x and 0, i.e. $x^+ = \max(x, 0)$, $x^- = \min(x, 0)$.
- $\lfloor x \rfloor$: integer part of real number x.
- $A^\top$: transpose of matrix A.
- $\det(A)$: the determinant of a square matrix A.
- $\mathrm{tr}(A)$: the trace of a square matrix A.
- $\mathrm{diag}(a)$: a square matrix with the vector a on the diagonal and zeros elsewhere.