$$
\begin{aligned}
\delta(t;i) &\triangleq \int \widehat{p}(t,x)(x-x_0)^i \left(\overline{f}_\epsilon(x)^2 - f_\epsilon(t,x)^2\right)\, dx \\
&= 2\epsilon \left(\int_0^T \frac{\partial f(s\epsilon^2, x_0)}{\partial x} w(s)\, ds - \frac{\partial f(t\epsilon^2, x_0)}{\partial x} \right) \\
&\quad \times \int \widehat{p}(t,x)(x-x_0)^{i+1}\, dx \\
&\quad + \epsilon^2 \left(\int_0^T \left[\frac{\partial f(s\epsilon^2, x_0)}{\partial x}\right]^2 w(s)\, ds - \left[\frac{\partial f(t\epsilon^2, x_0)}{\partial x}\right]^2 \right) \\
&\quad \times \int \widehat{p}(t,x)(x-x_0)^{i+2}\, dx \\
&\quad + \epsilon^2 \left(\int_0^T \frac{\partial^2 f(s\epsilon^2, x_0)}{\partial x^2} w(s)\, ds - \frac{\partial^2 f(t\epsilon^2, x_0)}{\partial x^2} \right) \\
&\quad \times \int \widehat{p}(t,x)(x-x_0)^{i+2}\, dx \\
&\quad + o(\epsilon^2).
\end{aligned}
$$

Calculating the integrals, we obtain to order $o(\epsilon^2)$,

$$
\begin{aligned}
\delta(t;0) &= \epsilon^2 v(t)^2 \left(\int_0^T \left[\frac{\partial f(s\epsilon^2, x_0)}{\partial x}\right]^2 w(s)\, ds - \left[\frac{\partial f(t\epsilon^2, x_0)}{\partial x}\right]^2 \right. \\
&\quad \left. + \int_0^T \frac{\partial^2 f(s\epsilon^2, x_0)}{\partial x^2} w(s)\, ds - \frac{\partial^2 f(t\epsilon^2, x_0)}{\partial x^2} \right), \\
\delta(t;1) &= 2\epsilon v(t)^2 \left(\int_0^T \frac{\partial f(s\epsilon^2, x_0)}{\partial x} w(s)\, ds - \frac{\partial f(t\epsilon^2, x_0)}{\partial x} \right), \\
\Delta(i) &= \frac{1}{2} \int_0^T \delta(t;i)\lambda(t)^2\, dt.
\end{aligned}
$$

For $w(t) = w_T(t)$, we obtain $\Delta(i) = 0$, $i = 0, 1$, and the theorem follows. □

Proposition 9.3.4 is proved by applying Theorem 9.A.1 to the equation (9.30). To compute $v(t)^2$, conditioning on $z(t)$ and using conditional independence of $X_0(t)$ and $z(t)$ we obtain,

$$
\begin{aligned}
\mathrm{E}\left((X_0(t)-x_0)^2 z(t) \right) &= \mathrm{E}\left(z(t)\mathrm{E}\left((X_0(t)-x_0)^2 \middle| z(\cdot) \right) \right) \\
&= \mathrm{E}\left(z(t) \int_0^t z(s)\lambda(s)^2\, ds \right) \\
&= \int_0^t \lambda(s)^2 \mathrm{E}\left(z(t)z(s) \right)\, ds.
\end{aligned}
\tag{9.99}
$$

Clearly